



Isospin-Driven Splitting of Chemical Potentials in Isobar Collisions from Lattice QCD

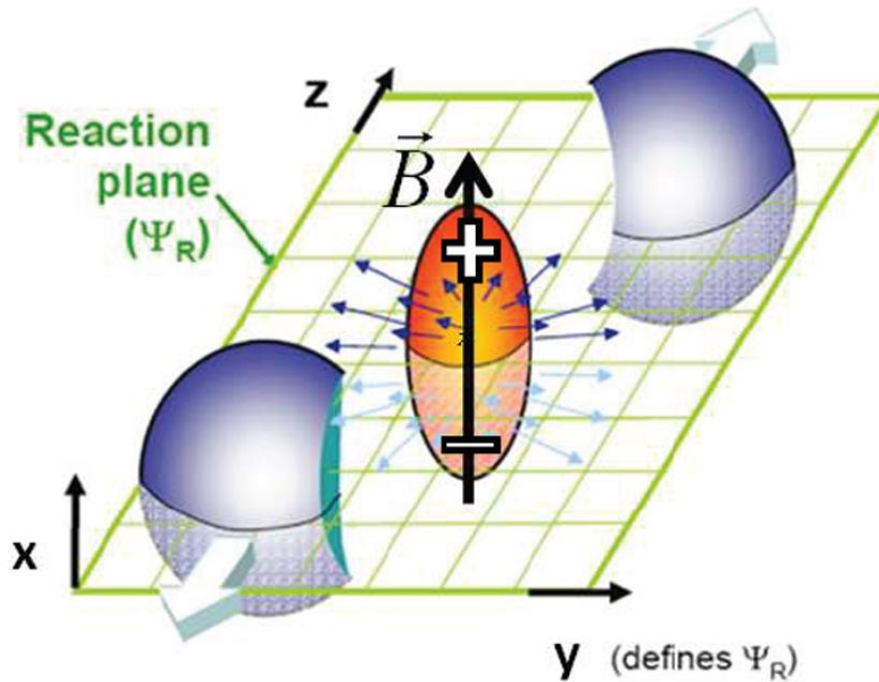
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In collaboration with Heng-Tong Ding, Jin-Biao Gu and Arpith Kumar
Based on arXiv:2606.30164 [hep-lat]

The 2nd Workshop on QCD Matter under Rotation and Magnetic Fields
Huainan · September 2026

Strong magnetic fields and the chiral magnetic effect

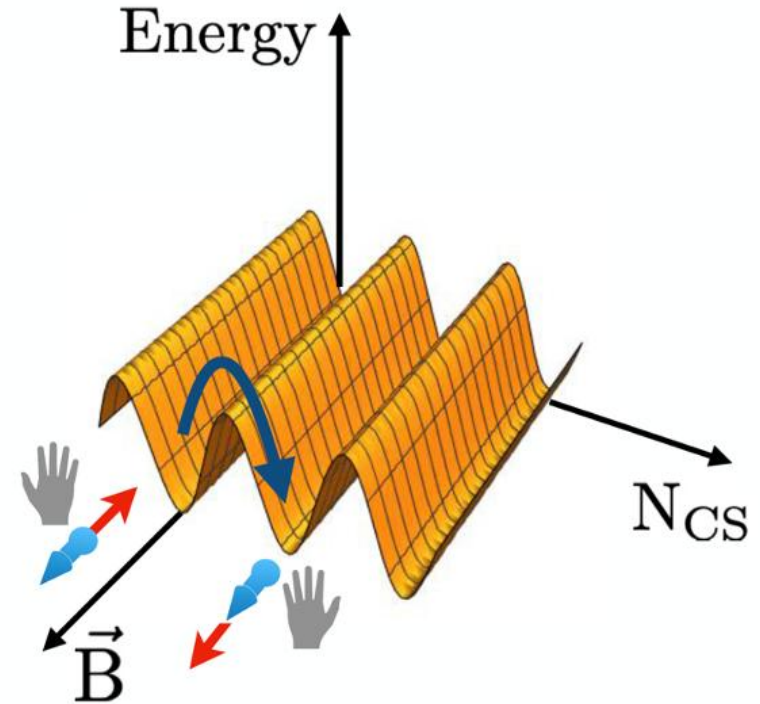


$10^{17} - 10^{18} \text{G}$ orders of Λ_{QCD}^2

V. Skokov, A. Illarionov, V. Toneev, IJMPA 24 (2009) 592

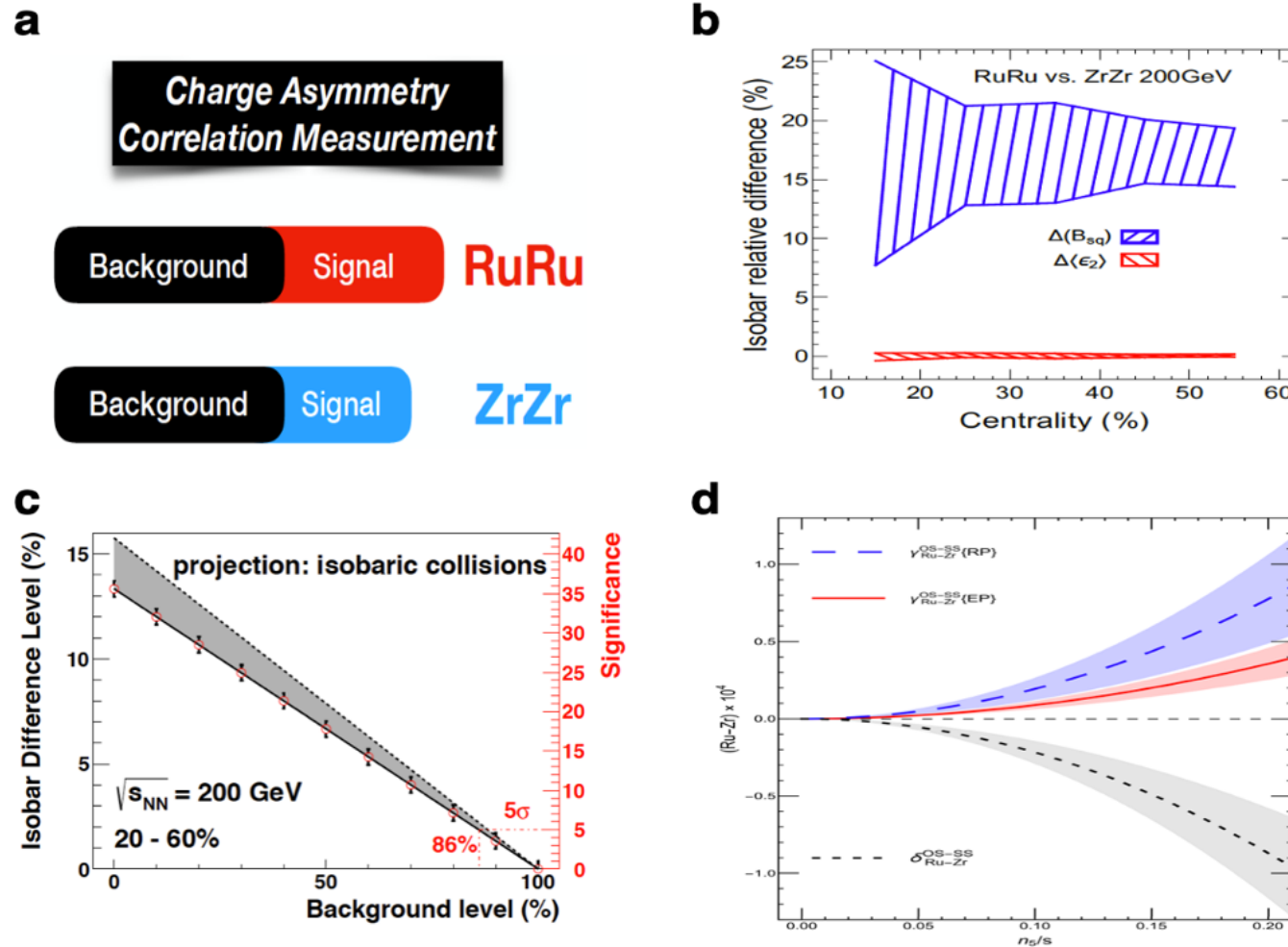
–RHIC $5m_\pi^2$ – LHC $70m_\pi^2$

W.-T. Deng, X.-G. Huang, PRC 85 (2012) 044907



D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3 (2021) 55 [2102.06623].

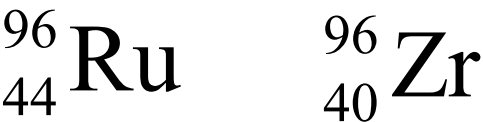
The isobar collision experiment



D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3 (2021) 55 [2102.06623].

The chemical-potential splittings in STAR isobar data

Nuclear composition



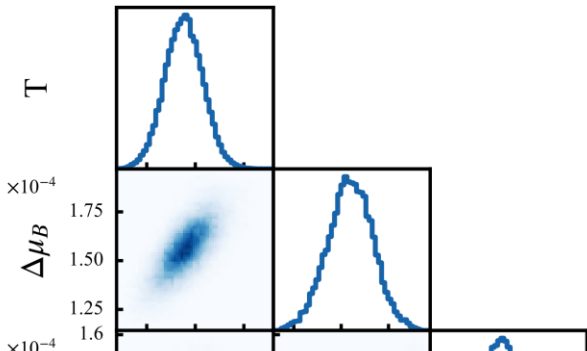
charge-to-baryon ratio

$$r \equiv \frac{n_Q}{n_B} \simeq \frac{Z}{A}$$
 $r_{\text{Ru}} \simeq 0.458$
 $r_{\text{Zr}} \simeq 0.417$

chemical potential splittings

$\Delta\mu_B, \Delta\mu_Q, \Delta\mu_S$

Relate to Zr-Ru Splitting Ratios in Isobar Experiments



$\Delta\mu_B$	$0.156^{+0.011}_{-0.012}$
$\Delta\mu_S$	$0.124^{+0.01}_{-0.01}$
$\Delta\mu_Q$	$-0.161^{+0.013}_{-0.012}$
$\frac{\Delta\mu_B}{\Delta\mu_Q}$	$-0.968^{+0.011}_{-0.012}$
$\frac{\Delta\mu_S}{\Delta\mu_Q}$	$-0.769^{+0.021}_{-0.026}$
	$0.59^{+0.028}_{-0.032}$

What is QCD benchmark for these isospin-driven chemical-potential splittings?

quality class

Bayesian analyses posterior

J. Grefa, C.Y. Tsang, R. Kumar, V. Dexheimer,C. Ratti, Z. Xu; arXiv:2601.21232 [nucl-th]

From chemical potential to coefficient q_1 and s_1

Taylor expansion $\hat{\mu}_{\{Q,S\}} \equiv \hat{\mu}_{\{Q,S\}}(T, eB, \hat{\mu}_B)$

$$\hat{\mu}_Q = q_1(r, T, eB) \hat{\mu}_B + q_3(r, T, eB) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

$$\hat{\mu}_S = s_1(r, T, eB) \hat{\mu}_B + s_3(r, T, eB) \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5)$$

Initial nuclei
constraint

$$n_Q / n_B = r, n_S = 0$$



$$q_1 = \frac{r(\chi_2^B \chi_2^S - \chi_{11}^{BS} \chi_{11}^{BS}) - (\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}{(\chi_2^Q \chi_2^S - \chi_{11}^{QS} \chi_{11}^{QS}) - r(\chi_{11}^{BQ} \chi_2^S - \chi_{11}^{BS} \chi_{11}^{QS})}$$

$$s_1 = -(\chi_{11}^{BS} + q_1 \chi_{11}^{QS}) / \chi_2^S$$



The fluctuations of conserved charge

$$\chi_{ijk}^{BQS}(T, eB) = \left. \frac{\partial^{i+j+k} (p / T^4)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\hat{\mu}_B = \hat{\mu}_Q = \hat{\mu}_S = 0}$$

Isospin-driven chemical-potential splitting ratios

The splittings of chemical potential:

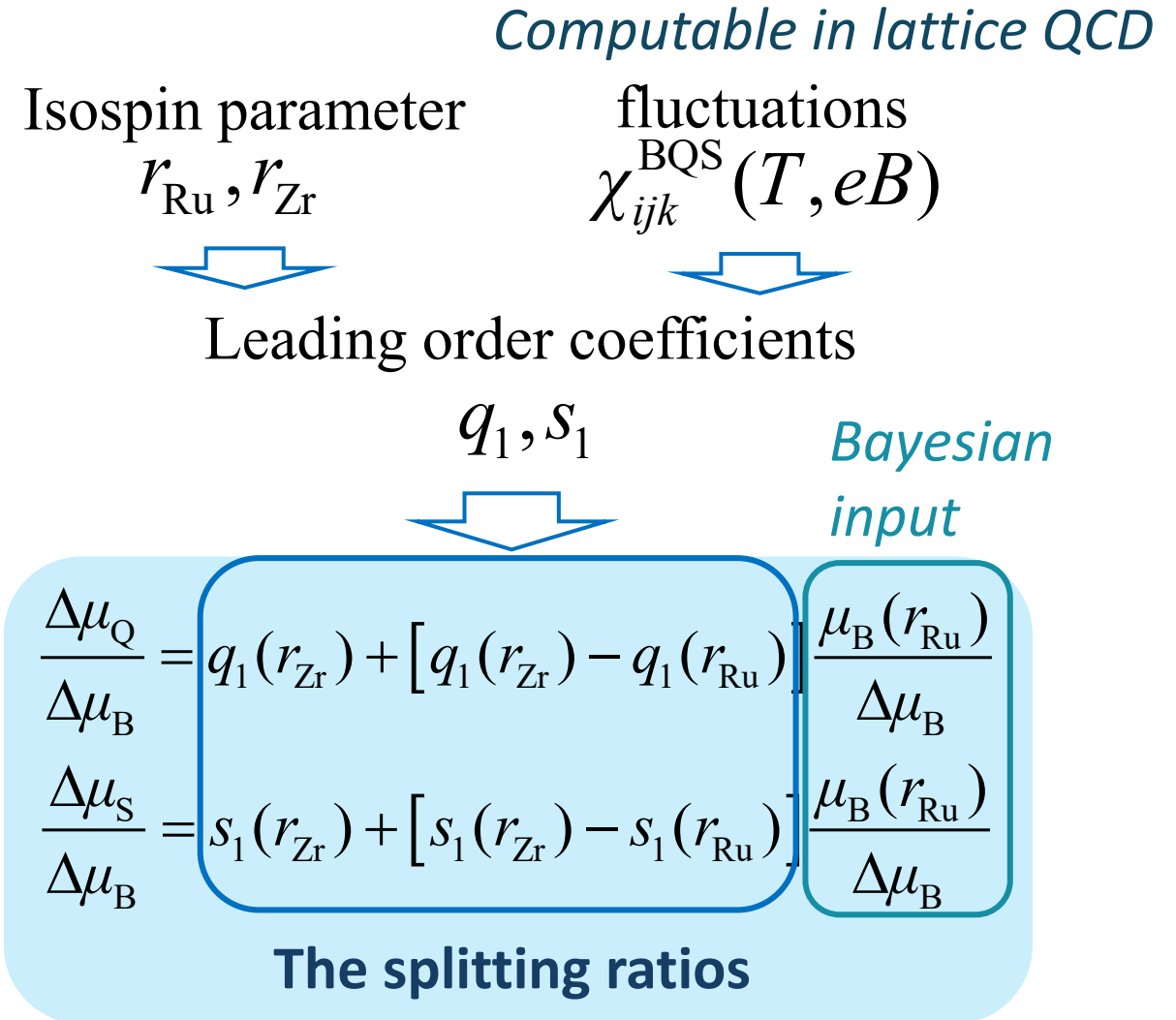
$$\Delta\hat{\mu}_X \equiv \hat{\mu}_X(r_{\text{Zr}}) - \hat{\mu}_X(r_{\text{Ru}}), \quad X \in \{\text{B}, \text{Q}, \text{S}\}$$



$$q_1 = \left(\frac{\mu_{\text{Q}}}{\mu_{\text{B}}}\right)_{\text{LO}}, s_1 = \left(\frac{\mu_{\text{S}}}{\mu_{\text{B}}}\right)_{\text{LO}}$$

$$\begin{aligned} \Delta\hat{\mu}_{\text{Q}} &\equiv q_1(r_{\text{Zr}})\hat{\mu}_{\text{B}}(r_{\text{Zr}}) - q_1(r_{\text{Ru}})\hat{\mu}_{\text{B}}(r_{\text{Ru}}), \\ &= q_1(r_{\text{Zr}})(\hat{\mu}_{\text{B}}(r_{\text{Ru}}) + \Delta\hat{\mu}_{\text{B}}) - q_1(r_{\text{Ru}})\hat{\mu}_{\text{B}}(r_{\text{Ru}}) \end{aligned}$$

$$\begin{aligned} \Delta\hat{\mu}_{\text{S}} &\equiv s_1(r_{\text{Zr}})\hat{\mu}_{\text{B}}(r_{\text{Zr}}) - s_1(r_{\text{Ru}})\hat{\mu}_{\text{B}}(r_{\text{Ru}}) \\ &= s_1(r_{\text{Zr}})(\hat{\mu}_{\text{B}}(r_{\text{Ru}}) + \Delta\hat{\mu}_{\text{B}}) - s_1(r_{\text{Ru}})\hat{\mu}_{\text{B}}(r_{\text{Ru}}) \end{aligned}$$



Lattice Setup

2+1 flavor QCD on Lattice

- Highly improved staggered fermions and a tree-level improved Symanzik gauge action
- physical pion mass : $m_l = m_s / 27$
- Lattice : $32^3 \times 8$ and $48^3 \times 12 \rightarrow \text{cont.est.}$

simulated T and eB range

- Temperature: along the $T_{pc}(eB)$ at finite eB
- Magnetic field : $0 \leq eB \lesssim 8M_\pi^2 \sim [0, 0.15] \text{ GeV}^2$

Isospin and magnetic-field dependence of q_1 and s_1

Evaluated along the
 $T = T_{pc}(eB)$:

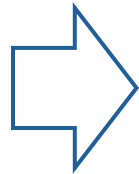
$$q_1(r, eB)$$

$$s_1(r, eB)$$



Isospin dependence at fixed eB

$$q_1(r), s_1(r) \quad \text{at} \quad eB = 0, 4, 8M_\pi^2$$



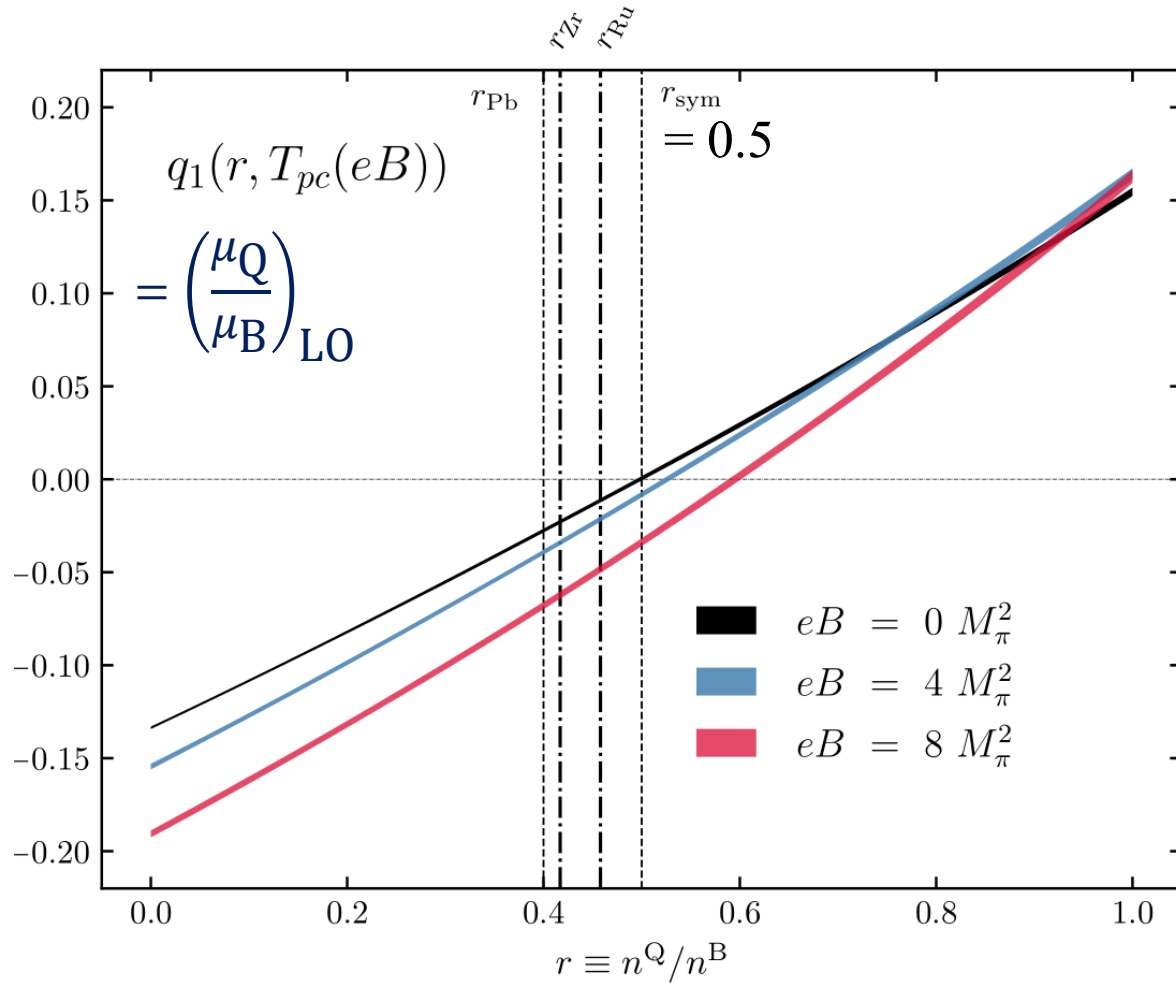
Magnetic field dependence at fixed r

$$q_1(eB), s_1(eB) \quad \text{at fixed} \quad r_{\text{Ru}}, r_{\text{Zr}}, r_{\text{Pb}}, r_{\text{sym}}$$

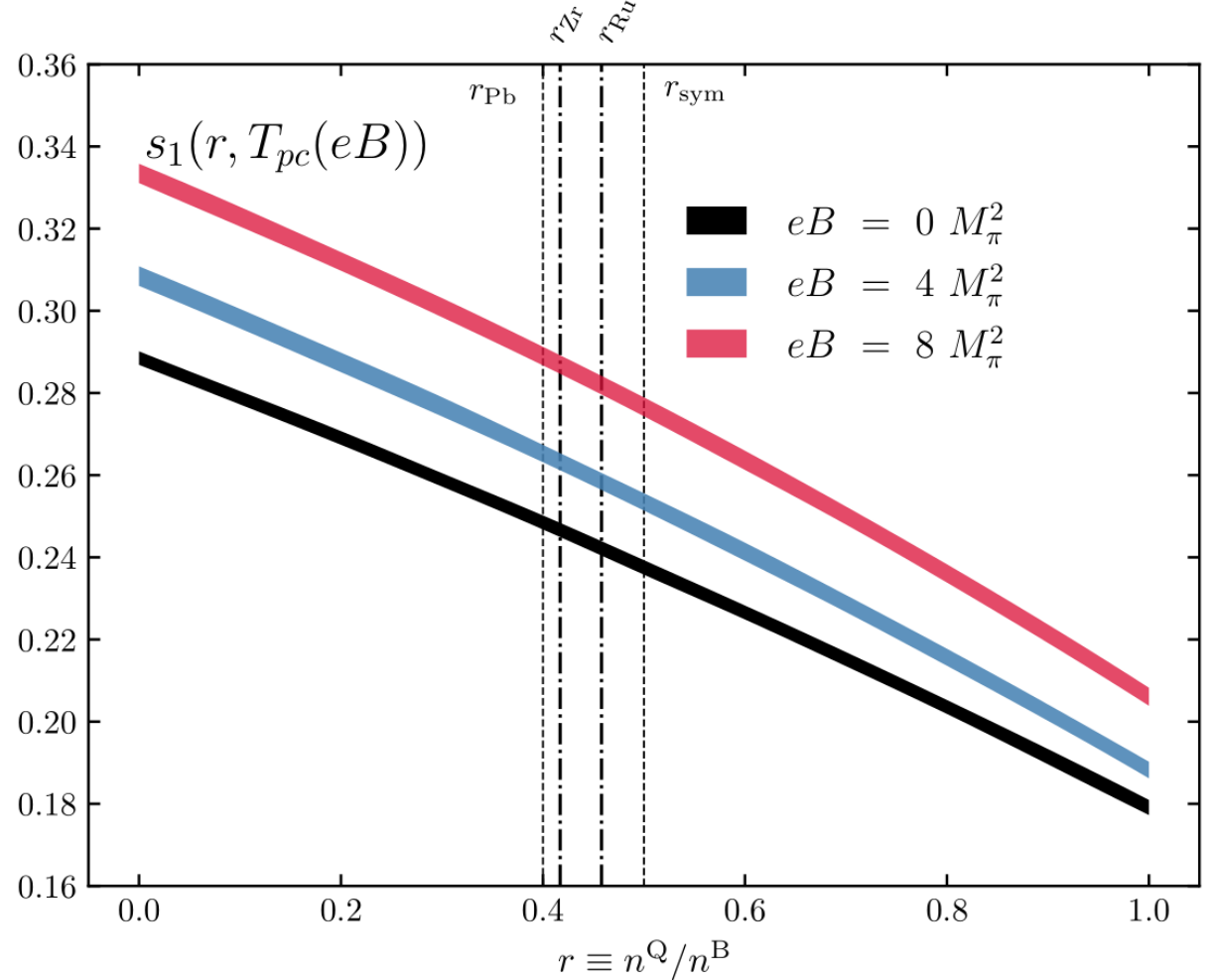
$$q_1(r_{\text{Zr}}, eB) - q_1(r_{\text{Ru}}, eB)$$

$$s_1(r_{\text{Zr}}, eB) - s_1(r_{\text{Ru}}, eB)$$

r –dependence of q_1 and s_1 along $T_{pc}(eB)$

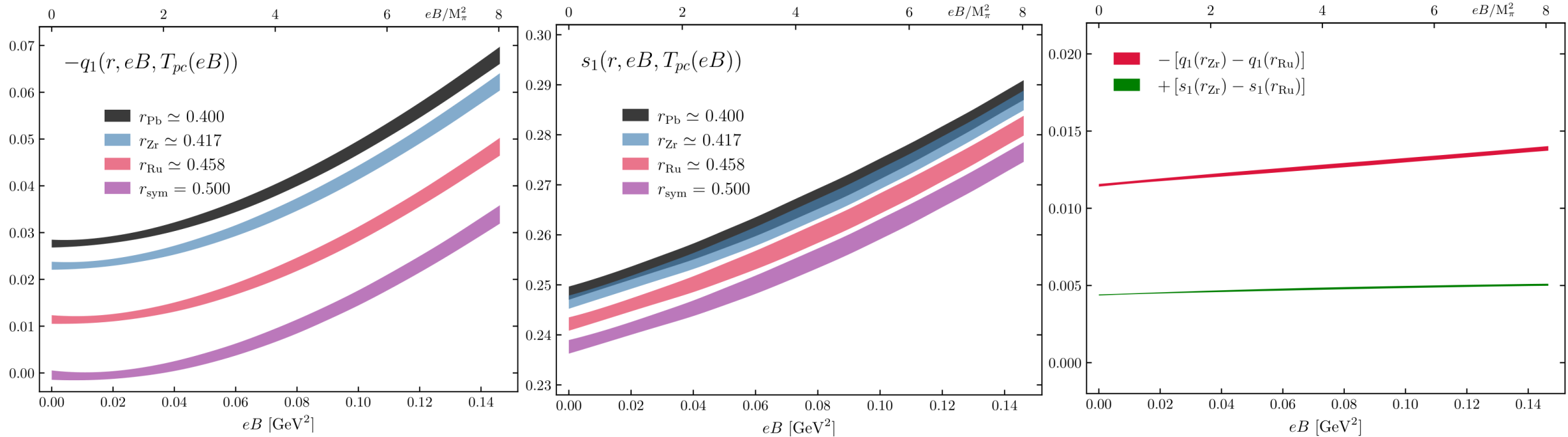


$$|q_1(r_{Pb})| > |q_1(r_{Zr})| > |q_1(r_{Ru})|$$



$$s_1(r_{Pb}) > s_1(r_{Zr}) > s_1(r_{Ru}) > 0$$

eB –dependence at fixed isospin parameter r



$$\frac{\Delta\mu_Q}{\Delta\mu_B} = q_1(r_{Zr}) + [q_1(r_{Zr}) - q_1(r_{Ru})] \frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$

$$\frac{\Delta\mu_S}{\Delta\mu_B} = s_1(r_{Zr}) + [s_1(r_{Zr}) - s_1(r_{Ru})] \frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$

Intercepts : $-q_1$ grows strongly, while s_1 grows mildly.

Slopes : $-\Delta q_1$ increases mildly, while Δs_1 is nearly eB -independent.

QMHRG baselines and STAR-motivated proxies

Fluctuations expressed in terms of hadron and resonance species R:

$$\chi_{ijk}^{\text{BQS}}(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \sum_R B_R^i Q_R^j S_R^k \frac{\partial^{i+j+k} p_R / T^4}{\partial \hat{\mu}_R^{i+j+k}} \Rightarrow \begin{matrix} q_1^{\text{QMHRG}} \\ s_1^{\text{QMHRG}} \end{matrix} \Rightarrow \left\{ \frac{\Delta\mu_Q}{\Delta\mu_B}, \frac{\Delta\mu_S}{\Delta\mu_B}, \frac{\Delta\mu_S}{\Delta\mu_Q} \right\}_{\text{QMHRG}}$$

R. Bellwied et al., Phys. Rev. D 101, 034506 (2020)

Proxy construction

$$\tilde{i} \equiv N_i - N_{\bar{i}}$$

$$\tilde{B} \equiv \tilde{p}$$

$$\tilde{Q}_{\text{PID}} \equiv \tilde{\pi} + \tilde{K}^+ + \tilde{p}$$

$$\tilde{S} \equiv \tilde{K}^+$$

Λ -extended

$$\tilde{B} \rightarrow \tilde{p} + \tilde{\Lambda}$$

$$\tilde{Q}_{\text{PID}} \equiv \tilde{\pi} + \tilde{K}^+ + \tilde{p}$$

$$\tilde{S} \rightarrow \tilde{K}^+ - \tilde{\Lambda}$$

STAR-motivated kinematic cuts

F. Karsch, K. Morita and K. Redlich,
|y| < 0.5, Phys. Rev. C 93 (2016) 034907

$$0.2 < p_T < 2.5 \text{ GeV} / c \quad \text{for } \pi, K,$$

$$0.45 < p_T < 2.5 \text{ GeV} / c \quad \text{for } p,$$

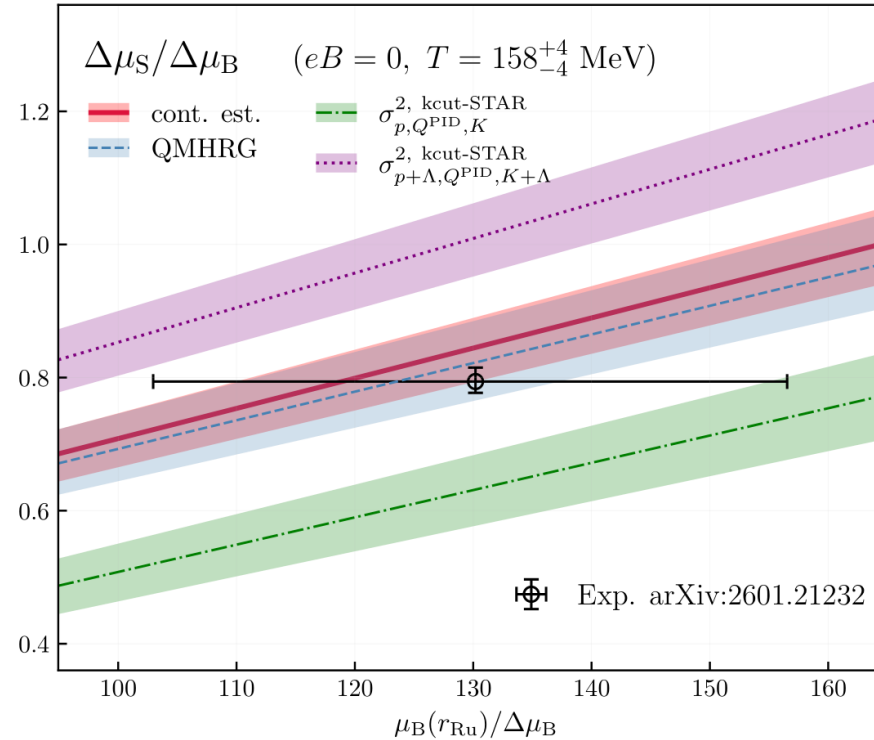
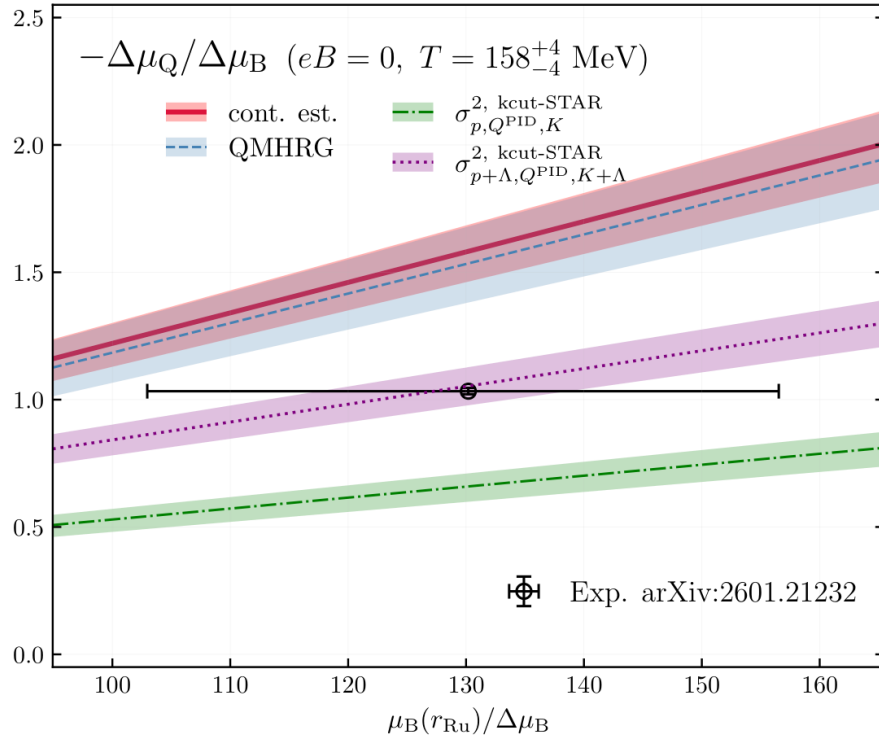
$$0.4 < p_T < 1.6 \text{ GeV} / c \quad \text{for } \Lambda.$$

STAR-cut proxy

$$\sigma_{p, Q^{\text{PID}}, K}^{2, \text{kcute-STAR}}$$

$$\sigma_{p+\Lambda, Q^{\text{PID}}, K+\Lambda}^{2, \text{kcute-STAR}}$$

Isospin-driven splitting ratios at vanishing magnetic field



The electric-charge ratio is displayed as $-\Delta\mu_Q/\Delta\mu_B$

Bayesian thermal-analysis extraction:

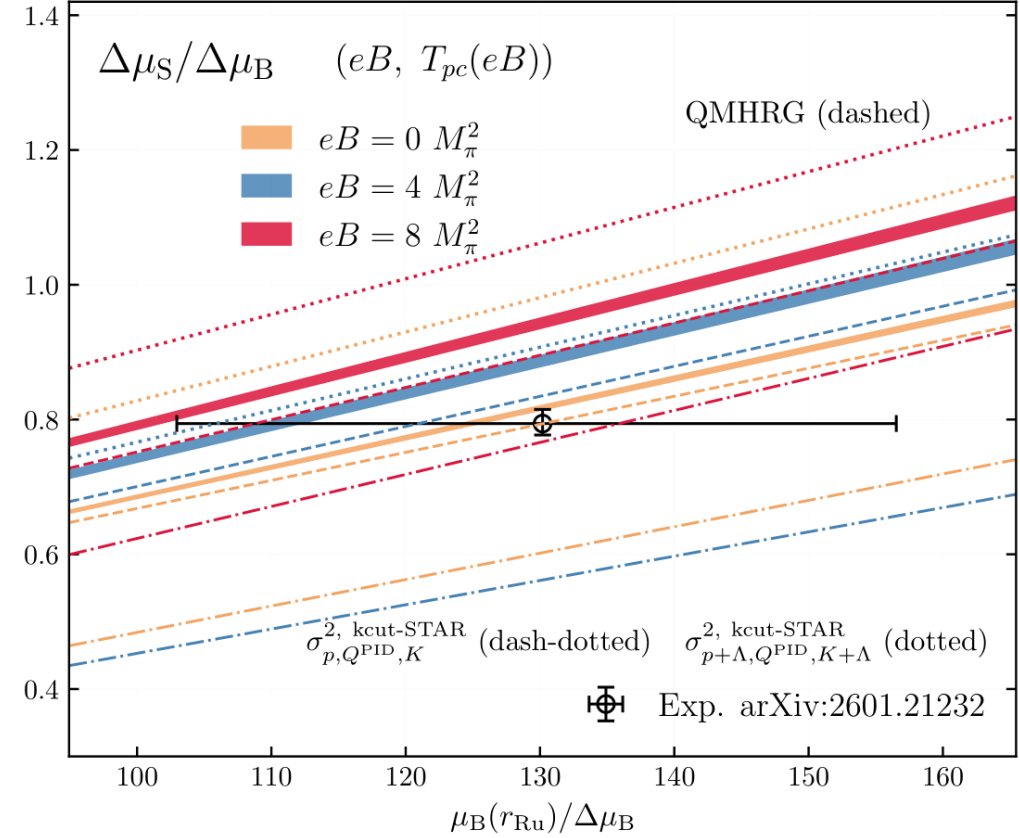
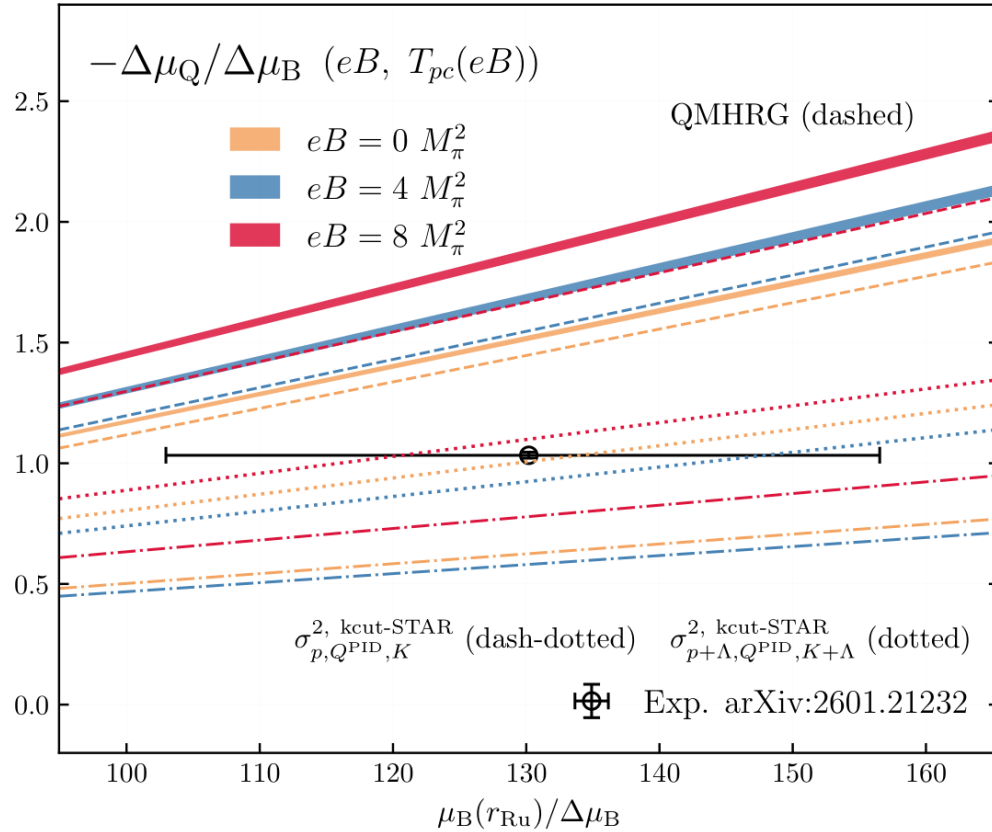
$$\mu_B(r_{\text{Ru}}) = 20.31^{+3.8}_{-4.0} \text{ MeV}$$

$$\Delta\mu_B = 0.156^{+0.011}_{-0.012} \text{ MeV}$$

$$\frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B} \simeq 130$$

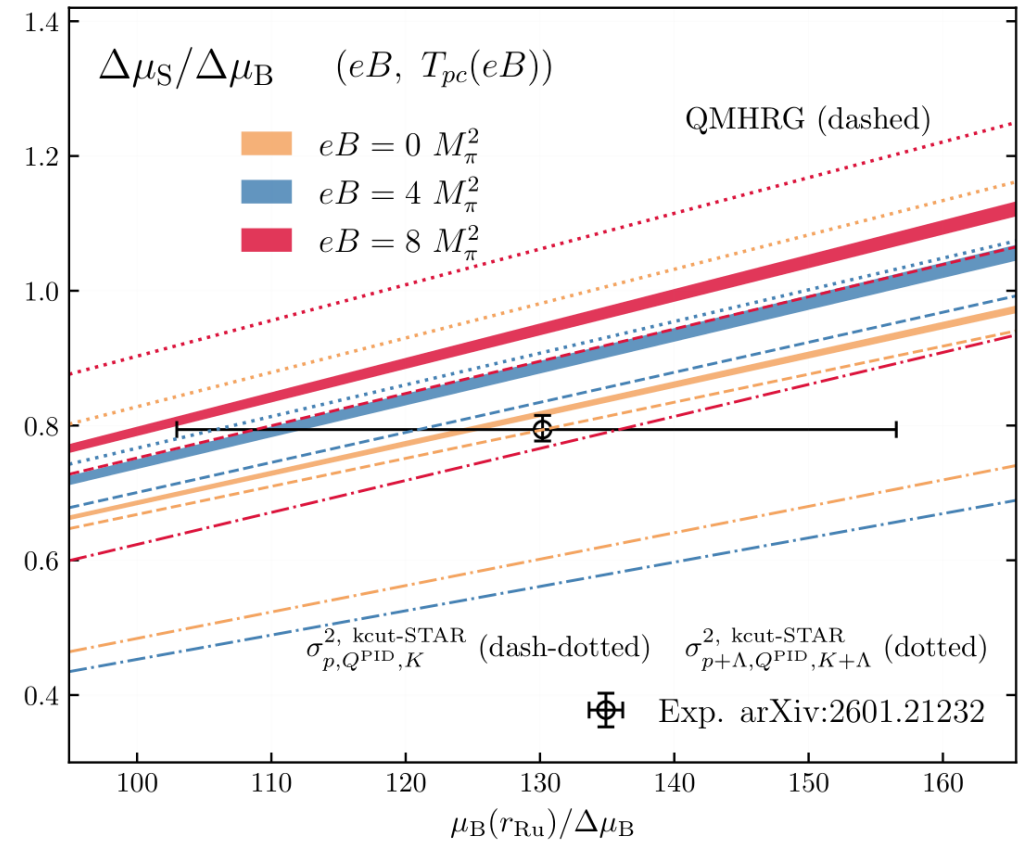
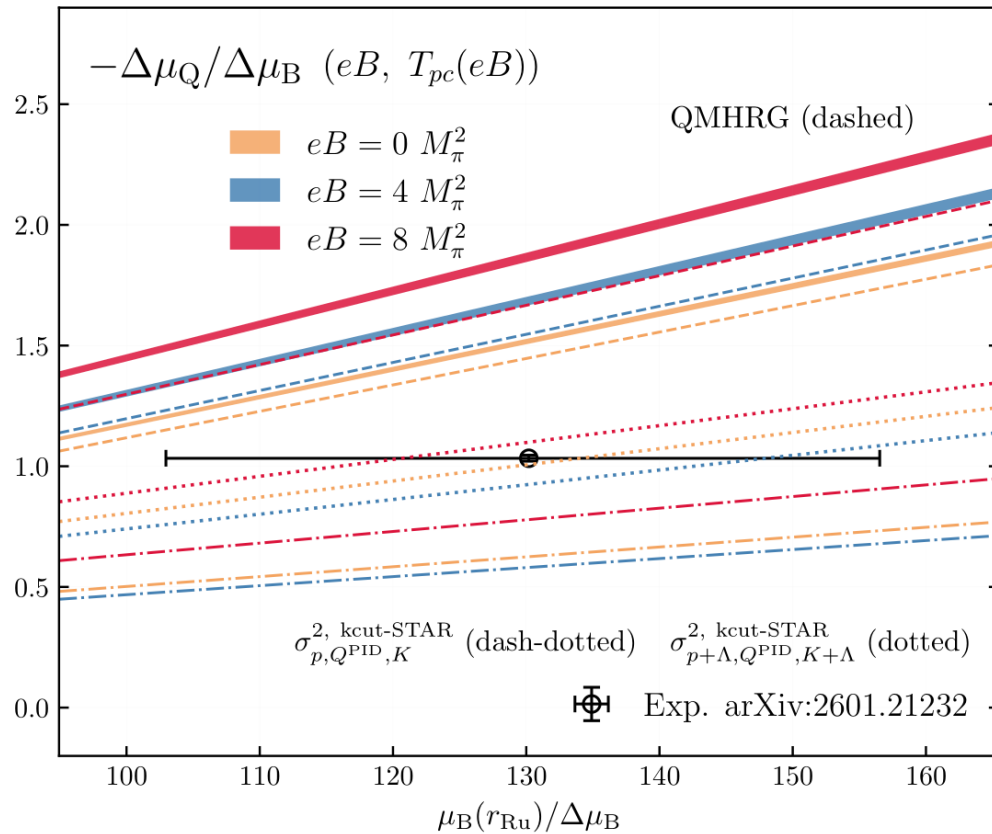
- LQCD : $\Delta\mu_Q < 0$, $\Delta\mu_S > 0$; closest Bayesian agreement in $\Delta\mu_S/\Delta\mu_B$.
- QMHRG : Follows the lattice results trend in the Bayesian window.
- Proxies : Restricted particle content and kinematic cuts produce sizable shifts.

Magnetic fields modify the splitting ratio



- Signs and monotonic $\mu_B(r_{Ru})/\Delta\mu_B$ -dependence are preserved at all eB .
- Increasing eB mildly enhances magnitude of the splitting ratios.
- QMHRG reproduce the behavior, while proxy with kinematic cut show the deviations.

Magnetic fields modify the splitting ratio



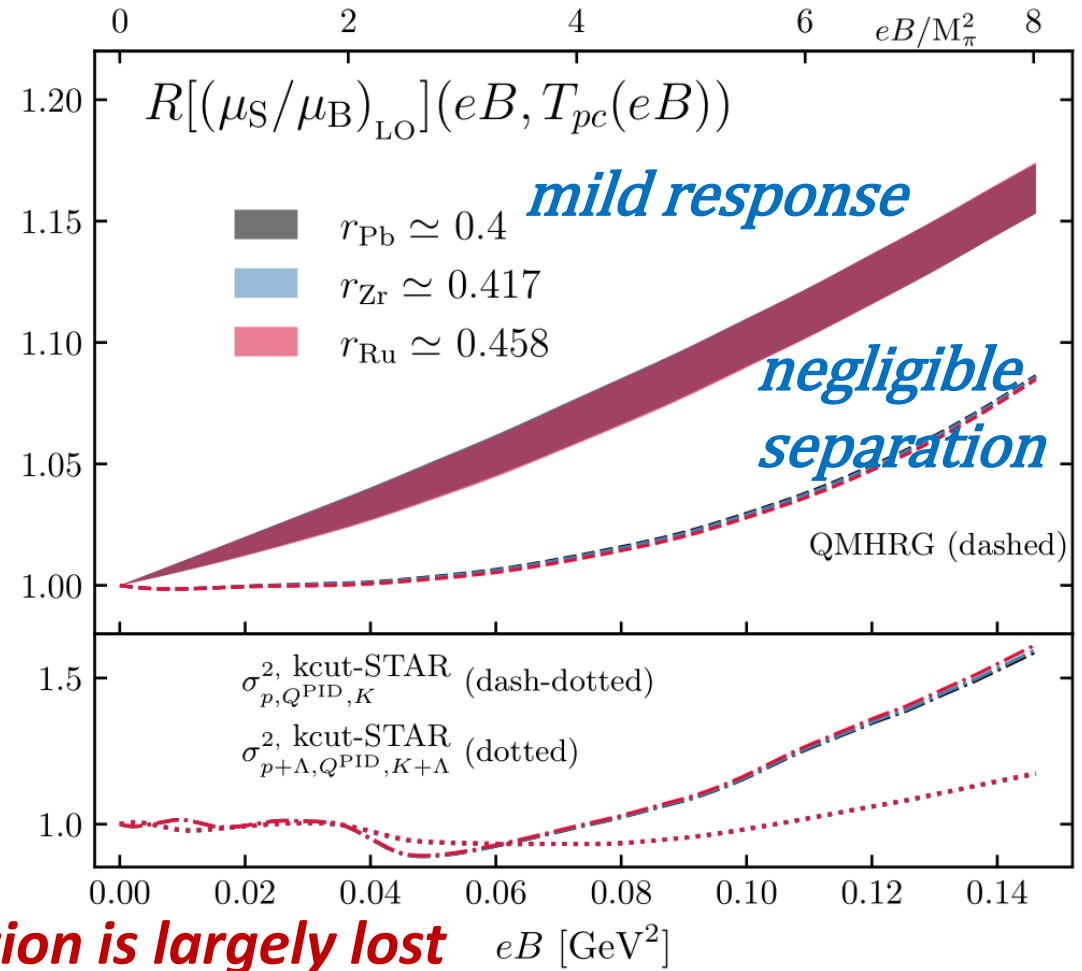
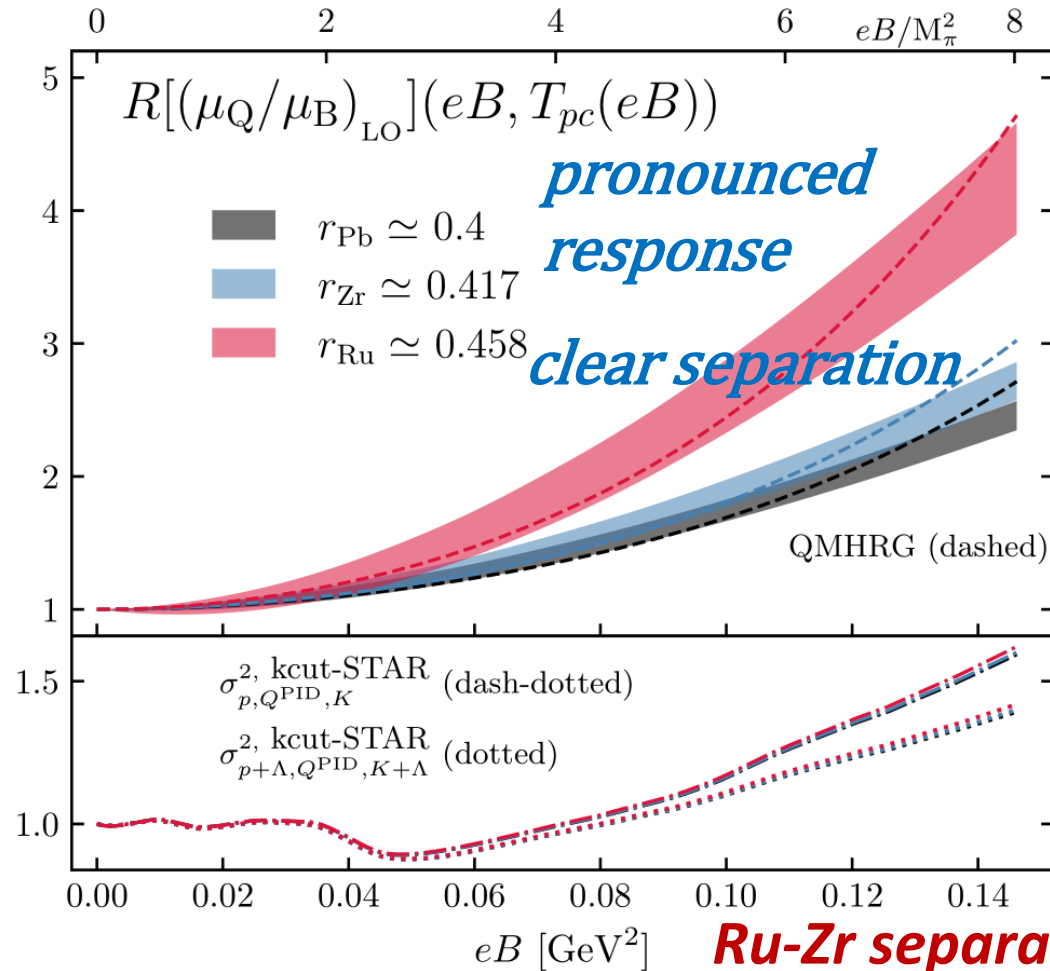
$$\frac{\Delta\mu_Q}{\Delta\mu_B} = q_1(r_{Zr}) + \left[q_1(r_{Zr}) - q_1(r_{Ru}) \right] \frac{\mu_B(r_{Ru})}{\Delta\mu_B}$$

Dominate, but weak eB-dependence

Can construct a sensitivity observable ?

Normalized ratios reveal a pronounced eB response

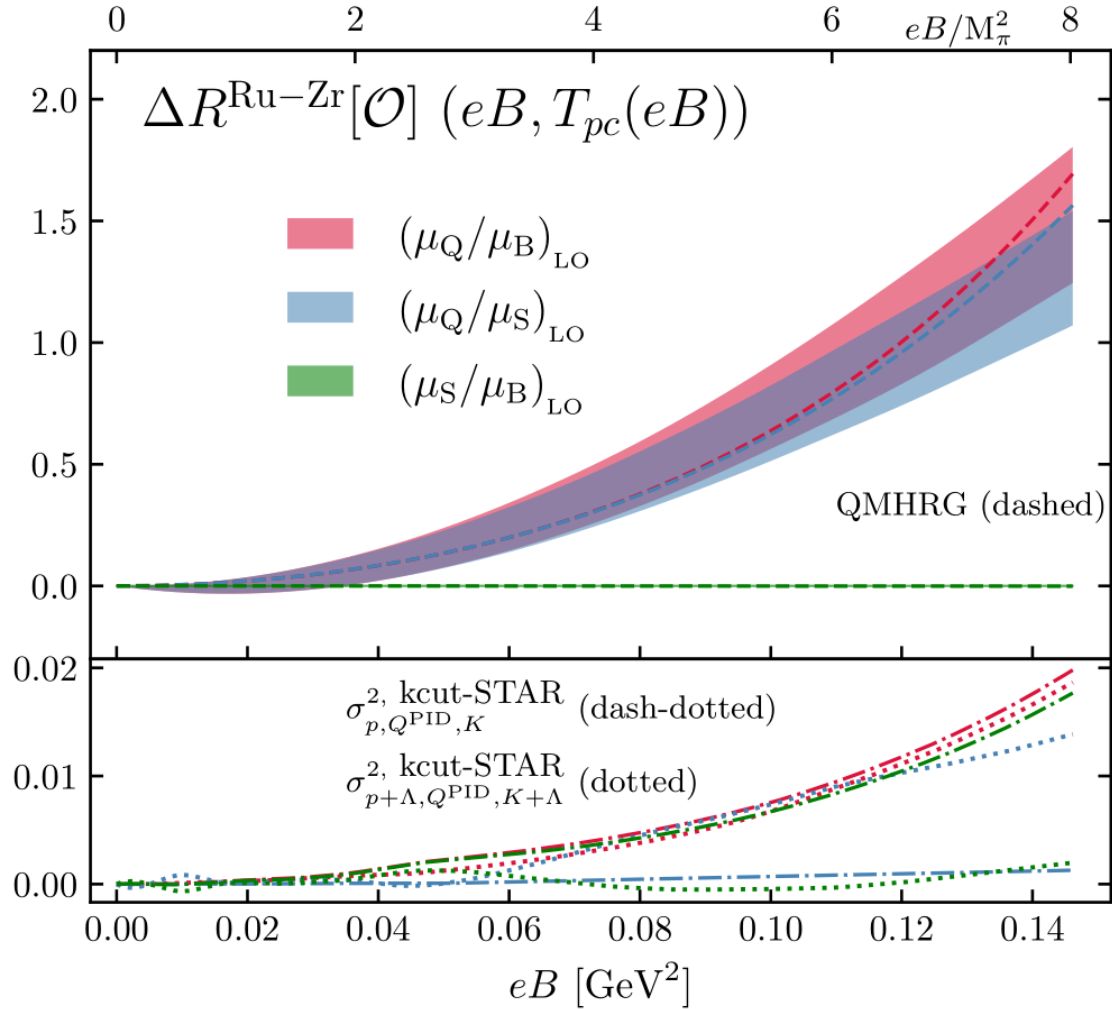
$$R[O](eB) \equiv \frac{O(r, eB, T_{pc}(eB))}{O(r, eB=0, T_{pc}(0))}, \text{ where } O \in \left\{ \left(\frac{\mu_Q}{\mu_B} \right)_{LO}, \left(\frac{\mu_S}{\mu_B} \right)_{LO} \right\}$$



Ru-Zr separation is largely lost

Ru-Zr difference of normalized magnetic responses

$\Delta R^{\text{Ru-Zr}}[O](eB) \equiv R_{\text{Ru}}[O](eB) - R_{\text{Zr}}[O](eB)$ positive values : stronger response in Ru than in Zr.



- ✓ $\Delta R^{\text{Ru-Zr}}[O]$: isolates the isospin dependence.
- ✓ LQCD/QMHRG: charge and mixed differences reach ~ 1.5 ; the strangeness difference stays near zero.
- ✓ Charged-hadron proxies largely wash out the Ru-Zr sensitivity, mainly due to the omission of neutral states such as neutrons and K^0 .

- Framework : q_1, s_1 , and their Ru-Zr differences determine the splitting ratios.
- At $eB = 0$: $\Delta\mu_Q < 0, \Delta\mu_S > 0$; $\Delta\mu_S/\Delta\mu_B$ agrees best with the Bayesian extraction.
- At $eB > 0$: Splitting-ratio magnitudes increase, while their slopes change only mildly.
- Normalized response : Pronounced Ru-Zr contrast in μ_Q -related ratios, but nearly none in μ_S/μ_B .
- Proxies : Overall eB response retained, Ru-Zr separation largely lost.

Thank you!

B pointing along the z direction:

$$u_x(n_x, n_y, n_z, n_\tau) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

No sign problem!

$$u_y(n_x, n_y, n_z, n_\tau) = \exp[iqa^2Bn_x]$$

$$u_z(n_x, n_y, n_z, n_\tau) = u_t(n_x, n_y, n_z, n_\tau) = 1$$

Quantization of the magnetic field

$$qB = \frac{2\pi N_b}{N_x N_y} a^{-2} \quad \longrightarrow \quad eB = \frac{6\pi N_b}{N_x N_y} a^{-2}$$

The periodicity of $U(1)$ links implies the constrain : $0 \leq N_b < N_x N_y / 4$

In this work, we restrict to $N_b \leq 6 \Leftrightarrow eB \simeq 8M_\pi^2$

Landau gauge

G.S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S.D. Katz, S. Krieg et al., JHEP 02 (2012) 044.

The NLO splitting ratio reads:

$$\left(\frac{\Delta\mu_Q}{\Delta\mu_B}\right)_{\text{NLO}} \equiv q_1(r_{\text{Zr}}) + (q_1(r_{\text{Zr}}) - q_1(r_{\text{Ru}})) \frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B} \quad \text{LO}$$
$$+ (q_3(r_{\text{Zr}}) - q_3(r_{\text{Ru}})) \hat{\mu}_B^2(r_{\text{Ru}}) \left(\frac{\mu_B(r_{\text{Ru}})}{\Delta\mu_B}\right) + 3q_3(r_{\text{Zr}}) \hat{\mu}_B^2(r_{\text{Ru}}) \quad \text{NLO}$$

Near the isobar freeze out point:

$$\hat{\mu}_B = \frac{20.31}{158} \simeq 0.129,$$
$$\hat{\mu}_B^2 \simeq 0.017 \simeq 2\%.$$

The lattice estimate:

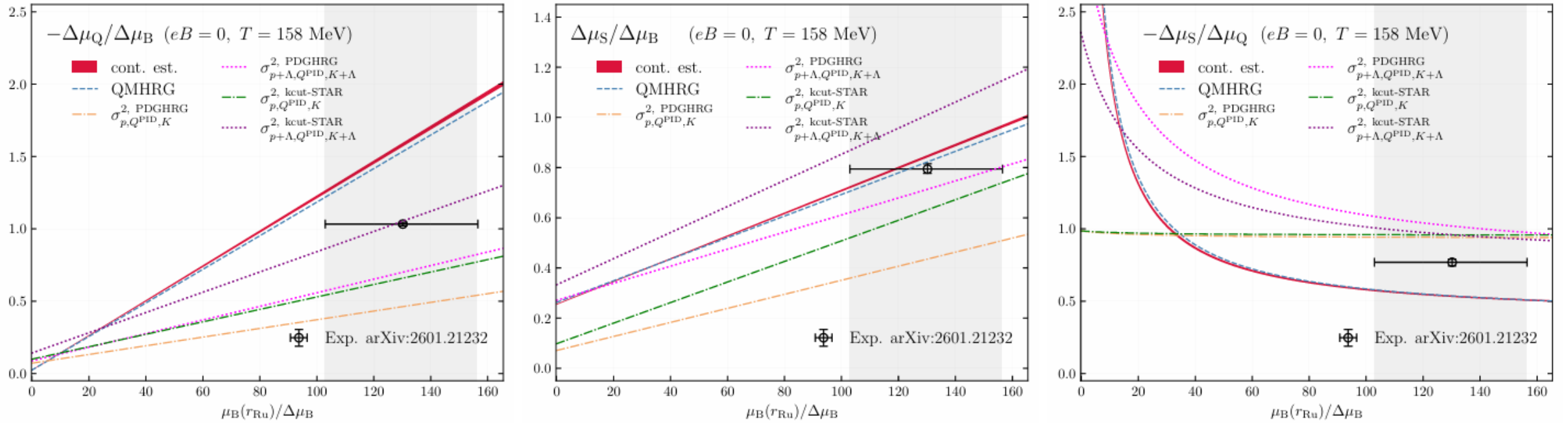
$$\frac{q_3}{q_1} \lesssim 2\%$$

H.-T. Ding, J.-B. Gu, A. Kumar, S.-T. Li and J.-H. Liu,
Phys. Rev. Lett. 132 (2024) 201903 [2312.08860].

- Suppresses the NLO corrections to $\Delta\mu_Q/\Delta\mu_B$ to $O(10^{-3})$ relative to the LO.
- Similarly for $\Delta\mu_S/\Delta\mu_B$, $\Delta\mu_S/\Delta\mu_Q$

Results in an extended $\mu_B(r_{Ru})/\Delta\mu_B$ window

BACKUP



- The charge and strangeness splitting ratios grow linearly with $\mu_B(r_{Ru})/\Delta\mu_B$, with the isospin-difference terms dominating in the experimental window.
- The mixed ratio begins to saturate near $1/2$ as the common $\mu_B(r_{Ru})/\Delta\mu_B$ dependence cancels.
- The Λ -extended proxy improves the strangeness sector at small $\mu_B(r_{Ru})/\Delta\mu_B$, but current proxies do not reproduce the lattice behavior in the experimental window.